

Pseudo-Riemannian cones with reducible holonomy group

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Abstract

By a classical theorem of Gallot (1979), a Riemannian cone $C(M)$ over a complete Riemannian manifold M is either flat or has irreducible holonomy. Since Killing spinors of M correspond to parallel spinors of $C(M)$, this result is important for the description of manifolds with Killing spinors. We consider some generalizations of this theorem to the case when M is a pseudo-Riemannian manifold. We prove that if M is a compact complete pseudo-Riemannian manifold and its cone $C(M)$ is decomposable (into a product of two pseudo-Riemannian manifolds) then the cone $C(M)$ is flat and M has constant curvature. We give a description of the local structure of a pseudo-Riemannian manifold M whose cone $C(M)$ has two complementary parallel distributions D^\pm . In particular, we show that if the cone $C(M)$ is not decomposable, then the manifold M has a para-Sasakian structure. For Lorentzian cones, we get a complete description of the possible (local) holonomy groups in terms of the metric of the base manifold M .

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