

IFWGP'07 | International Fall Workshop on Geometry and Physics

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Title: Uniqueness of the maximal helicoid

Abstract:

Recently Meeks and Rosenberg have proved that the classical helicoid is the unique properly embedded non flat simply connected minimal surface in R^3 .

The spacelike part H of the helicoid in R^3 is the unique surface having zero mean curvature in both Riemannian and Lorentzian ambient, besides spacelike planes. In other words, H is a maximal surface in the Lorent-Minkowski space L^3 and a minimal surface in the Euclidean space R^3 .

As surface in L^3 , H is a properly embedded maximal surface bounded by a regular lightlike curve of mirror symmetry. The last means that the harmonic maximal immersion X defining H extends harmonically to the mirror surface H^* by defining $X(p^*)=X(p)$. This property is also satisfied by Enneper's maximal surface, and no more examples with connected lightlike boundary of mirror symmetry are known.

The aim of this lecture is to sketch the following natural and deep uniqueness theorem:

Theorem. *The helicoid and Enneper's surface are the unique properly embedded maximal surfaces with connected lightlike boundary of mirror symmetry.*

Roughly speaking, the proof requires three ingredients:

- 1) Some new results about maximal graphs, specially the asymptotic properties of maximal graphs over a wedge and a finiteness theorem for maximal graphs with planar boundary and disjoint supports.
- 2) An adapted Colding-Minicozzi theory for maximal surfaces: the study of the associated blow down multigraph.
- 3) The control of the conformal structure and Weierstrass representation of the surface, following Meeks-Rosenberg ideas.