Constant curvature deformations of a metric admitting a Killing vector field

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Abstract

In ref. [1] it was showed that given an analytic semi-Riemannian metric g_{ab}, there always exist a 2-form F_{ab} and a scalar function α such that:

1. An arbitrary, previously chosen scalar constraint $\Psi(\alpha, F) = 0$ is satisfied and
2. The *deformed metric*
$$\overline{g}_{ab} := \alpha g_{ab} - \epsilon F^2_{ab}\,, \qquad , \tag{1}$$
where $\epsilon = \pm 1$ and $F^2_{ab} := F_{ac} g^{cd} F_{db}$, has constant curvature.

We shall refer to this result as the *deformation theorem.*

The proof is based on the existence of solutions to a certain partial differential system involving the Riemann tensor for $\overline{g}$. Since the Cauchy data are to be chosen on some non-characteristic hypersurface, there is enough freedom left so that the deformation (1) is by no means unique.

Let us now restrict ourselves to the case of a Lorentzian 4-manifold $\mathcal{V}$ and assume that the original metric g admits a Killing vector field X, i.e. $\mathcal{L}_X g = 0$. Is it possible to perform the deformation (1) so that X is also a Killing field for the deformed metric $\overline{g}$? We shall here answer affirmatively to this question provided that (i) X is not an isotropic vector, that is $g(X, X) \neq 0$, and (ii) the quotient space by the orbits of X, $\mathcal{S} := \mathcal{V}/X$, admits a manifold structure.

The requirement that the deformation (1) preserves the symmetries of g can then be used to reduce the wide non-uniqueness commented above.

We first study some algebraic features of the deformation (1), and then present an outline of a formalism introduced by Geroch [2] relating the metric g and the quotient metric h.

This permits to obtain a set of partial differential relations based on the requirements that $\overline{g}$ has constant curvature and admits the Killing field X. Some of these relations can be considered as the *reduced partial differential system* and the remaining ones as constraints on the Cauchy data.

Referències

[1] Llosa, J. and Soler, D. *Clas. Quantum Grav.* **22** (2005) 893

[2] Geroch, R., *J. Math. Phys.* **12** (1971) 918