Optimal control problems for affine connection control systems: characterization of extremals.

M. Barbero-Liñán M-C. Muñoz-Lecanda

Departamento de Matemática Aplicada IV, UPC, Barcelona, Spain.

Pontryagin's Maximum Principle [6] is considered as an outstanding achievement of the Optimal Control Theory. It has been used in a wide range of applications, such as medicine, traffic flow, robotics, economy, etc. Nevertheless, it is worth remarking that the Maximum Principle does not give sufficient conditions to compute an optimal trajectory; it only provides necessary conditions. Thus only candidates to be optimal trajectories, called extremals, are found. Maximum Principle gives rise to different kinds of them and, particularly, the so-called abnormal extremals have been studied because they can be optimal, as Liu and Sussmann, and Montgomery proved in subRiemannian geometry [4,5].

We build up a presymplectic constraint algorithm, similar to those defined in [2,3], to determine where the different kinds of extremals of an optimal control problem can be. After describing such an algorithm, we apply it to the study of extremals, specially the abnormal ones, in optimal control problems for affine connection control systems [1]. These systems model the motion of different types of mechanical systems such as rigid bodies, nonholonomic systems and robotic arms [1].

References

[1] F. Bullo, A. D. Lewis, *Geometric Control of Mechanical Systems. Modeling, analysis and design for simple mechanical control*, Texts in Applied Mathematics 49, Springer-Verlag, New York-Heidelberg-Berlin 2004.

[2] J. F. Cariñena, Theory of singular Lagrangians, *Fortschr. Phys.*, **38**(9)(1990), 641-679.

[3] M. J. Gotay, J. M. Nester, Presymplectic Lagrangian systems I: The constraint algorithm and the equivalence theorem, *Ann. Inst. H. Poincare Sect. A* **30**(2)(1979), 129-142.

[4] W. Liu, H. J. Sussmann, Shortest paths for sub-Riemannian metrics on rank-two distributions, *Mem. Amer. Math. Soc.* 564, Jan. 1996.

[5] R. Montgomery, Abnormal Minimizers, *SIAM J. Control Optim.*, **32**(6)(1994), 1605-1620.

[6] L. S. Pontryagin, V. G. Boltyanski, R. V. Gamkrelidze and E. F. Mischenko, *The Mathematical Theory of Optimal Processes*, Interscience Publishers, Inc., New York 1962.